Spring 2022 Math 208 M Midterm 2

NAME (First,Last) : ....

STUDENT ID ....

UW email ....

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** show your work and justify your answers.
- Your work needs to be neat and legible.

Problem 1 Let A be a 4×4 matrix with columns c_1, c_2, c_3, c_4. Suppose that by performing a sequence of elementary operations you can reduce A to

$$B = \begin{pmatrix} 1 & 5 & -1 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

1. Find the rank of A. No justification necessary.

rank $(A) = 3$

2. Find a basis for row(A), the row space of A.

$(1, 5, -1, 0)$
$(0, 1, 1, -2)$
$(0, 0, 1, 2)$

3. Find a basis for Null(A), the null space of A. Show your work.

$x_3 + 2x_4 = 0 \rightarrow x_3 = -2x_4$

$x_2 + x_3 - 2x_4 = 0 \rightarrow x_2 = -x_3 + 2x_4 = 4x_4$

$x_1 + 5x_2 - x_3 = 0 \rightarrow x_1 = -5x_2 + x_3 = -22x_4$

General solution of $Ax = 0$ in vector form is:
$(-22x_4, 4x_4, -2x_4, x_4) = x_4(-22, 4, -2, 1)$

Basis for Null(A) is $(-22, 4, -2, 1)$ (any non zero scalar multiple of this vector would do)

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4. Is c_1, c_2, c_4 a basis for col(A), the column space of A ? Justify your answer.

Yes if we look at $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ This matrix is in reduced echelon form and it has 3 pivots so its columns are linearily independent

This means $c_1\, c_2\, c_4$ are 3 linearly independent vectors in col(A) and since rank A = 3 They form a basis for col A

Consider $T : R^4 \to R^4$, $T(\vec{v}) = A\vec{v}$, where A is the matrix from the previous page.

1. Is T onto ? Justify your answer.

No because rank(A) = 3 so col(A) has dimension 3 and therefore it is not equal to R^4

2. Is T one to one ? Justify your answer.

No by unifying theorem if T is not onto it cannot be one to one

OR

No because Null(A) $\neq \{0\}$ (see part 3)

or consider that nullity(A) = 4 - 3 = 1 $\neq 0$)

Problem 2 This problem has two unrelated parts.

Find the matrices for 2 different linear transformations T_1 and T_2: $R^3 \to R^2$ both having the values $T_1((1,0,0)) = T_2((1,0,0)) = (2,-1)$ and
$T_1((0,1,1)) = T_2((0,1,1)) = (1,0)$ or explain why this is not possible.

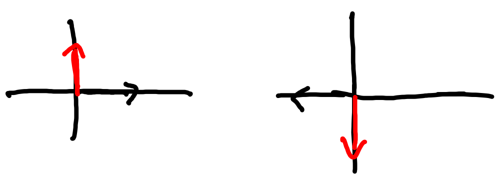
The matrix M of a linear transformation from R^3 to R^2 has for columns $T\begin{pmatrix}1\\0\\0\end{pmatrix}$ $T\begin{pmatrix}0\\1\\0\end{pmatrix}$ $T\begin{pmatrix}0\\0\\1\end{pmatrix}$

we know that $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$ suppose $T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$

then $T\begin{pmatrix}0\\0\\1\end{pmatrix} = T\left(\begin{pmatrix}0\\1\\1\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}a\\b\end{pmatrix} = \begin{matrix}1-a\\-b\end{matrix}$

so any matrix of the form $\begin{bmatrix}2 & a & 1-a\\-1 & b & -b\end{bmatrix}$ would work. We could have the matrix for T_1, $A = \begin{bmatrix}2 & 0 & 1\\-1 & 0 & 0\end{bmatrix}$ and the matrix for T_2 $B = \begin{bmatrix}2 & 1 & 0\\-1 & 0 & 0\end{bmatrix}$ (many other answers are possible)

Find the matrix of the linear transformation $T : R^2 \to R^2$ that rotates a vector (x,y) 180 degrees. Show your work to explain how you found this matrix..



we want $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$

$T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$

So $M = \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$

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Problem 3 This problem has two unrelated parts.

1. Give an example of a 3x3 matrix A such that Null(A)=span $\left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)$, and col(A)=span $\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$, or explain why this is not possible.

$A = \begin{bmatrix} 0 & 0 & 0 \\ a & b & c \\ 0 & 0 & 0 \end{bmatrix}$ This tells us col(A)= span $\begin{pmatrix}0\\1\\0\end{pmatrix}$
not all of $a, b, c = 0$

We also need $A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\a\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ so $a = 0$

$A\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\b\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$: $\cdot 0$

$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ would work

2. Give an example of a 3x3 invertible matrix A such that col(A)=span $\left(\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}\right)$, or explain why this is not possible.

Not possible, if col A = span $\left(\begin{pmatrix}1\\0\\1\end{pmatrix} \begin{pmatrix}2\\2\\1\end{pmatrix}\right)$
rank A = 2 not 3 so A is not invertible